

Particle Heating by Alfvenic Turbulence in Hot Accretion Flows

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ABSTRACT

Recent work on Alfvenic turbulence by Goldreich & Sridhar (1995; GS) suggests that the energy cascades almost entirely perpendicular to the local magnetic field. As a result, the cyclotron resonance is unimportant in dissipating the turbulent energy. Motivated by the GS cascade, we calculate the linear collisionless dissipation of Alfven waves with frequencies much less than the proton cyclotron frequency, but with perpendicular wavelengths of order the Larmor radius of thermal protons. In plasmas appropriate to hot accretion flows (proton temperature \gg electron temperature) the dissipated Alfven wave energy primarily heats the protons. For a plasma with $\beta \lesssim 5$, however, where β is the ratio of the gas pressure to the magnetic pressure, the MHD assumptions utilized in the GS analysis break down before most of the energy in Alfven waves is dissipated; how the cascade then proceeds is unclear.

Hot accretion flows, such as advection dominated accretion flows (ADAFs), are expected to contain significant levels of MHD turbulence. This work suggests that, for $\beta \gtrsim 5$, the Alfvenic component of such turbulence primarily heats the protons. Significant proton heating is required for the viability of ADAF models. We contrast our results on particle heating in ADAFs with recent work by Bisnovatyi-Kogan & Lovelace (1997).

Subject headings: accretion – hydromagnetics – plasmas – turbulence

1. Introduction

In astrophysical accretion flows with significant angular momentum, the accreting gas is believed to form either an optically thick, geometrically thin, disk (Shakura & Sunyaev 1973; see Frank, King, & Raine 1992 for a review) or an optically thin, geometrically thick, quasi-spherical flow (Shapiro, Lightman, & Eardley 1976; Ichimaru 1977;

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Rees et al. 1982; Narayan & Yi 1994, 1995a, 1995b; Abramowicz et al. 1995). In thin accretion disks, the gas cools so efficiently that all of the viscously generated energy is radiated locally. The resulting low temperature implies that the mean free path due to Coulomb collisions is a small fraction of the size of the disk. Consequently, a purely fluid description of the accreting gas is reasonable. By contrast, the high temperatures in hot accretion flows entail that the (field-free) mean free path is often comparable to the size of the accretion flow. Collective plasma effects are thus likely to be significant (Rees et al. 1982).

In hot accretion flows, the plasma is usually assumed to be two temperature, with the ions significantly hotter than the electrons (Shapiro, Lightman, & Eardley 1976). Such a temperature difference is only possible if the ions and electrons are thermally decoupled. For low accretion rates ($\dot{m} \lesssim \alpha^2$, where \dot{m} is the accretion rate in Eddington units and α is the Shakura-Sunyaev viscosity parameter) Coulomb collisions are too inefficient to force a one temperature plasma (Rees et al. 1982). One of the outstanding plasma physics problems relevant to accretion theory is whether there are collective effects which transfer energy from the ions to the electrons on a timescale short compared to the inflow time of the gas, thus invalidating the two temperature assumption (e.g., Phinney 1981). The only such mechanism that we are aware of in the astrophysical literature, due to Begelman & Chiueh (1988), is probably not efficient enough to eliminate the two temperature nature of the flow (Narayan & Yi 1995b). In this paper we thus assume that the two-temperature formalism is valid and that the only thermal coupling between electrons and ions is due to Coulomb collisions.

Recently, there has been substantial work (Narayan & Yi 1994, 1995a, 1995b; Abramowicz et al. 1995; Chen et al. 1995; Nakamura et al. 1997) on a class of hot optically thin accretion solutions first discovered by Ichimaru (1977), advection-dominated accretion flows (ADAFs). The defining characteristic of an ADAF is that it is underluminous for its accretion rate (i.e., $L \ll \dot{M}c^2$). This arises because “standard” ADAF models assume that, by virtue of their larger mass, most of the viscously generated energy heats the ions. Since only a small fraction of this energy is transferred to the electrons via Coulomb collisions, the total energy radiated (almost all by the electrons) is much less than the total energy generated by viscosity (Ichimaru 1977; Rees et al. 1982). The remaining viscously generated energy is stored as thermal energy of the ions and is advected onto the central object.

The assumption that viscosity heats the ions is crucial for the relevance of ADAF models. This enables ADAF solutions to exist so long as the electrons and ions are thermally decoupled ($\dot{m} \lesssim \alpha^2$). If viscosity were to predominantly heat the electrons, (optically thin) ADAF solutions would only exist when the electron cooling time is longer than the inflow time of the gas, which occurs provided $\dot{m} \lesssim 10^{-4}\alpha^2$ for synchrotron cooling (Mahadevan & Quataert 1997). Thus, in the context of ADAFs, the issue of which particles receive the viscous energy acquires particular importance. In this paper,

we present a preliminary investigation of this question. In §6.5 we discuss some related work by Bisnovatyi-Kogan & Lovelace (1997). We note that an investigation similar to ours, but with somewhat different conclusions, was carried out independently by Gruzinov (1997).

We assume that the energy generated by viscosity is initially converted into large scale MHD waves. For simplicity, we focus primarily, but not exclusively, on Alfvén waves. The large scale waves cascade to smaller wavelengths until they are dissipated. The relative heating of ions and electrons is determined by which particle species is primarily responsible for the dissipation of the waves. We focus on linear collisionless dissipation mechanisms, as these should be of principle importance in hot accretion flows (§6).

Collisionless dissipation of MHD waves and the back reaction of the dissipated energy on the electron and proton distribution functions has been considered extensively as a mechanism for accelerating particles in solar flares (e.g., Melrose 1994; Miller & Roberts 1995; Miller, LaRosa & Moore 1996). These ideas have also recently been applied to particle acceleration in accretion disk corona (Dermer, Miller & Li 1996; Li, Kusunose, & Liang 1996; Li & Miller 1997). Following the seminal work of Kraichnan (1965), most of these calculations assume that the turbulent cascade is isotropic, that is, that the turbulent energy density at any scale depends only on the magnitude of the wavevector, and is independent of its direction with respect to the mean magnetic field. Recent work on incompressible MHD turbulence by Goldreich & Sridhar (1995; 1997) and Sridhar & Goldreich (1994), collectively referred to hereafter as GS, suggests that this assumption is inapplicable for Alfvénic turbulence. This is discussed in more detail in the next section (§2).

We use the GS turbulent cascade to motivate the parameter regime (in wavevector space) in which we investigate collisionless dissipation of Alfvén waves; we also briefly discuss the collisionless dissipation of other MHD modes (§3). In §4 we qualitatively discuss the response of the electron and proton distribution functions to the dissipation of waves in a nearly perpendicular Alfvénic cascade (we do not, however, solve the quasi-linear equations). Throughout we concentrate on plasmas for which the ion temperature is greater than (often much greater than) the electron temperature and the gas pressure is comparable to, or greater than, the magnetic pressure, a regime rarely explored in calculations of the dissipation of MHD waves. In §5 we apply our calculations to the GS cascade and address two difficult, but important, questions: (1) to what extent is the linear analysis of §3 applicable to the strong Alfvénic cascade developed by GS and (2) is the dissipation found in §3 strong enough to dissipate the turbulent energy before the MHD assumptions used in the GS analysis are invalid. In §6 we apply our results to hot, two temperature, accretion flows, in particular ADAFs, and in §7 we summarize our results. We have attempted to make §6, which contains our astrophysical applications, comprehensible without a detailed understanding of the plasma physics calculations in

§3-§5. Towards this end, Appendix A contains definitions of a number of quantities used repeatedly in this paper.

2. Alfvenic Turbulence

Energy injected into a fluid/plasma on large spatial scales, if it is unable to dissipate, builds up to nonlinear amplitudes and cascades to smaller wavelength (larger wavevector) perturbations; this continues until dissipation becomes important and the turbulent energy is converted into thermal energy. To investigate the range of wavelengths where dissipation occurs (the “dissipation range” of the turbulence), two characteristics of the nonlinear cascade are particularly important. The first is the cascade time, i.e., the time for nonlinear effects to transfer energy from a wavevector $\sim \mathbf{k}$ to a wavevector $\sim 2\mathbf{k}$. This determines how rapid the dissipation must be to halt the cascade. The second is the path of the cascade in wavevector space. Does it depend only on $|\mathbf{k}| \equiv k$ or also on the direction? If the dissipation is a function of \mathbf{k} (and not just k), as it is for MHD modes, this distinction is crucial.

If there is no preferred direction in the fluid, the turbulence is isotropic, i.e., just a function of k . For MHD turbulence, however, the local magnetic field picks out a direction and so isotropy is not guaranteed. In fact, numerical simulations have long shown that incompressible MHD turbulence is anisotropic, with the energy cascading primarily perpendicular to the mean magnetic field (e.g., Shebalin et al. 1983). Incompressible MHD turbulence corresponds roughly to cascading Alfven waves, since both the fast and slow MHD modes are compressive. Recent work on Alfvenic turbulence has clarified the nature of this perpendicular cascade (GS; Montgomery & Matthaeus 1995; Ng & Bhattacharjee 1996).

Linear Alfven waves satisfy the dispersion relation $\omega = v_A k_z$, where ω is the mode frequency, v_A is the Alfven speed and k_z is the component of the wavevector along the mean magnetic field (taken to be along the z-direction). GS argue that Alfvenic turbulence naturally evolves into a “critically balanced” state in which the cascade time at a scale \mathbf{k} is comparable to the linear wave period at that scale. Furthermore, the parallel and perpendicular sizes of a wave at any scale are correlated, with $k_z \sim k_\perp^{2/3} L^{-1/3}$, where L is the outer scale of the turbulence (the scale on which energy is injected). The fluctuating magnetic field strength on any scale is given by $B_k \sim B_{out}(k_\perp L)^{-1/3}$, where B_{out} is the excitation amplitude on the outer scale.¹

¹The GS analysis breaks down if the fluxes of turbulent energy parallel and anti-parallel to the mean magnetic field are not equal. This precludes application of their theory to the solar wind, but this assumption should be applicable in hot accretion flows.

In the next two sections we calculate properties of the linear dissipation of nearly perpendicular Alfvén waves ($k_\perp \gg k_z$). This calculation is motivated by, but does not explicitly utilize, the Alfvénic cascade of GS. Because Alfvén “waves” in the GS cascade only live for \sim a mode period before nonlinear effects transfer their energy to smaller spatial scales, it is somewhat misleading to speak of them as waves (the turbulence is strong rather than weak in the turbulence theory sense). We will generally treat this notion as unproblematic, but in §5, having discussed the results of the linear analysis, we will return to the issue of its applicability to the GS cascade.

3. Linear Collisionless Dissipation of MHD Waves in Two Temperature Plasmas

3.1. Qualitative Considerations

Waves in a magnetized plasma can in general have electric and magnetic fields both perpendicular and parallel to the mean magnetic field. These fields can strongly effect the motion of particles through resonant interactions. This occurs when the frequency of the wave, in the frame moving with the particle along the field line, is an integer multiple of the particle’s cyclotron frequency,

$$\omega - k_z v_z = n\Omega, \quad (1)$$

where v_z is the particle’s velocity along the magnetic field and Ω is the relativistic cyclotron frequency (e.g. Melrose 1980). When this condition is satisfied, the particle and wave are in phase and the wave can efficiently accelerate the particle. In a collision dominated plasma, however, such phase coherence is impossible to maintain.

In the MHD limit, $\omega \ll \Omega_p$ and $k\rho_p \ll 1$, where ρ_p is the Larmor radius of protons with the thermal speed and Ω_p is the proton cyclotron frequency; in this limit the Alfvén wave dispersion relation is $\omega = k_z v_A$ and the resonance condition becomes

$$v_A - v_z = nv_A\Omega/\omega. \quad (2)$$

For $\omega \ll \Omega_p$, $n \neq 0$ resonances in equation (2) can only be satisfied by particles with $v_z \sim nv_A\Omega/\omega \gg v_A$. Since the thermal speeds of particles are typically of order the Alfvén speed, there are a negligible number of such particles; $n \neq 0$ resonances are consequently unimportant.

For $n = 0$, resonance occurs when the wave’s phase speed along the field line, $v_\parallel = \omega/k_z$, equals v_z . Particles with $v_z \lesssim v_\parallel$ are accelerated by the wave, while those with $v_z \gtrsim v_\parallel$ are decelerated. Thus, the wave is damped provided that the slope of

the particle distribution function at $v_z = v_{\parallel}$ is negative – as it is for a Maxwellian. A necessary (but not sufficient) condition for strong damping is that v_{\parallel} be comparable to the thermal speed of the particles, so that there are a large number of resonant particles.

The $n = 0$ resonance actually corresponds to two physically distinct wave-particle interactions. In Landau damping (LD), particle acceleration is due to the longitudinal electric field perturbation of a wave (i.e., the usual electrostatic force, E_z). In transit-time damping (TTD), the magnetic analogue of LD, the interaction is between the particle’s effective magnetic moment ($\mu = mv_{\perp}^2/2B$) and the wave’s longitudinal magnetic field perturbation, B_z (Stix 1992). TTD is thus analogous to Fermi acceleration (for discussions of the relationship between the two, see Achterberg 1981; Miller 1991).

In the MHD limit, which corresponds to small wavevectors, the Alfvén wave has both $E_z = 0$ and $B_z = 0$ and so is undamped by linear collisionless effects. For larger wavevectors, the MHD approximations are less applicable and kinetic theory corrections to E_z and B_z become important, leading to finite dissipation of the Alfvén wave; the $n \neq 0$ resonances *may* become important, if the wavevector has a significant component parallel to the background magnetic field. For the perpendicular cascade of Alfvén waves due to GS, however, when $k_{\perp}\rho_p \sim 1$, $\omega \sim \Omega_p(\rho_p/L)^{1/3} \ll \Omega_p$ and so $n \neq 0$ resonances can be satisfied only by particles with $v_z \gg v_A$, of which there are a negligible number. Since we expect significant E_z and/or B_z , and thus significant dissipation, when $k_{\perp}\rho_p \sim 1$, this implies that, even in the dissipation range, only $n = 0$ resonances are important.

We now investigate in detail the collisionless dissipation of Alfvén waves, focusing on $k_{\perp}\rho_p \sim 1$ and $k_z\rho_p \ll 1$. This problem has been considered by a number of authors (e.g., Akhiezer et al. 1975; Hasegawa & Chen 1976; Stefani 1976), but not in the parameter regime of interest to us.

3.2. Detailed Calculations: Methods

Consider a collisionless hydrogen plasma which is homogeneous, fully ionized, and threaded by a mean magnetic field, $\mathbf{B} = B_0\hat{\mathbf{z}}$. We assume that, in the unperturbed state, each particle species in the plasma (electrons and protons) has an isotropic, non-relativistic, thermal distribution function with no bulk (average) velocities.² Small amplitude perturbations to the equilibrium state of the plasma satisfy the following dispersion relation, which is obtained by linearizing and Fourier transforming (in time and space) Maxwell’s equations (Stix 1992; Chapter 1)

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \frac{\omega^2}{c^2} \epsilon \cdot \mathbf{E} = 0. \quad (3)$$

²By isotropic we mean that the temperature is the same perpendicular and parallel to $\hat{\mathbf{z}}$.

\mathbf{E} is the electric field perturbation and the dielectric tensor, ϵ , is given by $\epsilon_{ij} = \delta_{ij} + \sum_s \chi_{ij}^s$, where the sum is over the susceptibility tensor (χ_{ij}^s) of each particle species in the plasma; for our case, s takes on two values, e and p, for electrons and protons, respectively. The susceptibility tensor is calculated by combining the linearized and Fourier transformed versions of Maxwell's equations and the collisionless Boltzmann equation, and is given by, taking \mathbf{k} in the x-z plane, (Stix 1992; Chapter 10)

$$\begin{aligned}
\chi_{xx}^s &= \frac{m_p v_{tp}}{m_s v_{ts}} \frac{c^2}{v_A^2} \frac{\exp(-\lambda_s)}{\tilde{\omega} \lambda_s \eta} \sum_{n=-\infty}^{\infty} n^2 I_n Z_o(\xi_n) \\
\chi_{xy}^s &= -\chi_{yx}^s = \frac{m_p v_{tp}}{m_s v_{ts}} \frac{-ic^2}{v_A^2} \frac{\exp(-\lambda_s)}{\tilde{\omega} \eta} \sum_{n=-\infty}^{\infty} n(I_n - I'_n) Z_o(\xi_n) \\
\chi_{yy}^s &= \frac{m_p v_{tp}}{m_s v_{ts}} \frac{c^2}{v_A^2} \frac{\exp(-\lambda_s)}{\tilde{\omega} \eta} \sum_{n=-\infty}^{\infty} \left[\frac{n^2}{\lambda_s} I_n + 2\lambda_s (I_n - I'_n) \right] Z_o(\xi_n) \\
\chi_{xz}^s &= \chi_{zx}^s = \frac{q_s}{|q_s|} \frac{\sqrt{2}c^2}{v_A^2} \frac{\sqrt{\lambda_p} \exp(-\lambda_s)}{\tilde{\omega} \eta \lambda_s} \sum_{n=-\infty}^{\infty} n I_n [1 + \xi_n Z_o(\xi_n)] \\
\chi_{yz}^s &= -\chi_{zy}^s = \frac{q_s}{|q_s|} \frac{i\sqrt{2}c^2}{v_A^2} \frac{\sqrt{\lambda_p} \exp(-\lambda_s)}{\tilde{\omega} \eta} \sum_{n=-\infty}^{\infty} (I_n - I'_n) [1 + \xi_n Z_o(\xi_n)] \\
\chi_{zz}^s &= \frac{m_p v_{tp}^2}{m_s v_{ts}^2} \frac{2c^2}{v_A^2} \frac{\exp(-\lambda_s)}{\tilde{\omega} \eta^2} \sum_{n=-\infty}^{\infty} (\tilde{\omega} - n\Omega_s/\Omega_p) I_n [1 + \xi_n Z_o(\xi_n)], \tag{4}
\end{aligned}$$

where $v_{ts} = (2k_B T_s/m_s)^{1/2}$, q_s , m_s , and T_s are the thermal speed, charge, mass, and temperature of the particles, respectively. $v_A = B_0/(4\pi\rho)^{1/2}$ is the Alfvén speed and I_n is the modified Bessel function with argument $\lambda_s = k_\perp^2 v_{ts}^2 / 2\Omega_s^2 = 0.5k_\perp^2 \rho_s^2$, where $\rho_s = v_{ts}/\Omega_s$ is the Larmor radius of particles with the thermal velocity and $\Omega_s = q_s B_0/m_s c$ is the non-relativistic cyclotron frequency (taken to be a signed quantity). Prime denotes differentiation with respect to λ_s . Note that we are primarily interested in two temperature plasmas so that T_p and T_e are, in general, not equal.

In deference to the problem of interest, we measure mode wavelengths and frequencies in terms of the proton Larmor radius and cyclotron frequency, using $\eta = k_z \rho_p$ and $\lambda_p = 0.5k_\perp^2 \rho_p^2$ for the parallel and perpendicular components of the wavevector and $\tilde{\omega} = \omega/\Omega_p$ for the mode frequency. We emphasize, however, that equations (3) and (4) are valid regardless of the magnitude of η , λ_p , or $\tilde{\omega}$, and can be used to investigate the properties of any plasma waves (subject to the validity of the assumptions stated at the beginning of this subsection).

In equation (4), Z_o , the plasma dispersion function with argument

$$\xi_n = \frac{\omega - n\Omega_s}{v_{ts} k_z}, \tag{5}$$

is given by, taking $k_z \geq 0$, (Stix 1992; Chapter 8)

$$Z_o(\xi) = \frac{1}{\sqrt{\pi}} \int_{\Gamma} dz \frac{\exp(-z^2)}{z - \xi}, \quad (6)$$

where the contour, Γ , is such that the pole at $z = \xi$ lies above the contour of integration in the complex z plane. Mathematically, collisionless dissipation arises from the contribution of this pole to the susceptibility tensor, which is $\propto \exp(-\xi^2)$ by the residue theorem. Thus, a necessary (but, again, not sufficient) condition for strong damping is $\xi_n \lesssim 1$, which is the thermal average of the single particle resonance condition, equation (1).

Solving the dispersion relation yields, for a given real \mathbf{k} , a complex mode frequency, whose imaginary part, γ , represents the growth or dissipation of the wave. It does not, however, directly reveal the relative energy absorbed by the protons and electrons as the wave is damped. Strictly speaking, this can only be obtained from a nonlinear theory since energy considerations are necessarily second order in the amplitude. The Ohmic heating law, however, gives that the rate of change of energy of particle species s is $\propto \mathbf{j}_s \cdot \mathbf{E}$, where \mathbf{j}_s is the current. The complication is that the Ohmic heating law includes both the increasing thermal energy of the resonant particles (responsible for the damping) and the decreasing oscillation energy of the non-resonant particles (e.g., Barnes 1968a). When considering heating of the plasma by wave dissipation, one wishes to calculate only the former, not the latter. In the weak damping limit ($\gamma/\omega \ll 1$), this can be obtained from the Ohmic heating law by relating \mathbf{j}_s to \mathbf{E} using the susceptibility χ *evaluated at real frequencies*. This yields (Barnes 1968b; Stix 1992)

$$P_s = \frac{\mathbf{E}^* \cdot \chi_s^a|_{\text{Im}(\omega)=0} \cdot \mathbf{E}}{4W}, \quad (7)$$

where P_s is the energy absorbed in a mode period, per unit wave energy, by particle species s and $\chi_s^a = (\chi_s - \chi_s^\dagger)/2i$ is the antihermitian part of the susceptibility tensor. Physically, χ_s^a is evaluated at real frequencies since this entails that its only contribution is from the imaginary part of $Z_o(\xi_n)$; in turn, for $\text{Im}(\omega) = 0$, the only contribution to $\text{Im}(Z_o)$ is from the poles at $z = \xi_n$. As discussed below equation (6), the contributions from the poles in Z_o correspond to the thermal average of the single particle resonance condition (eq. [1]). Setting $\text{Im}(\omega) = 0$ in equation (7) therefore isolates the contribution from the resonant interactions, which is precisely what one wishes to do when determining particle heating. In equation (7), W , the wave energy, is given by

$$W = \frac{1}{16\pi} [|\mathbf{B}|^2 + \mathbf{E}^* \cdot \frac{\partial}{\partial \omega}(\omega \epsilon_h) \cdot \mathbf{E}], \quad (8)$$

where $\epsilon_h = (\epsilon + \epsilon^\dagger)/2$ is the hermitian part of the dielectric tensor and \mathbf{B} is the wave's magnetic field perturbation. We note that, for $\gamma/\omega \ll 1$, energy conservation implies that $P_p + P_e = 2\gamma T$, where $T \equiv 2\pi/\text{Re}(\omega)$ is the mode period. For the dissipation of Alfvén waves of interest to us, this is well satisfied even if $\gamma T \sim 1$; we therefore always use P_p and P_e as estimates of the proton and electron heating rates, respectively.

3.3. Alfven Waves with large k_{\perp}

Because equation (3) has an infinite number of roots (mostly strongly damped waves with no fluid counterparts), some care must be taken in its solution. Our technique is to first solve the dispersion relation in a simple limit (e.g., the MHD limit and plasma parameters such that the wave is weakly damped); this ensures that we know which mode we are investigating. We then incrementally change the wavevector and/or the plasma parameters and follow the properties of the solution. We emphasize that we have used the exact form of the susceptibility tensor, with no approximations (save for terminating the sum over Bessel functions at some appropriately large number).

In Figure 1 we show several properties of the Alfven wave as a function of T_p/T_e and the parameter $\lambda_p = 0.5k_{\perp}^2\rho_p^2$, which measures the perpendicular wavelength of the wave; the results correspond to a $\beta = 1$ plasma, where β , the ratio of the gas pressure to the magnetic pressure, is given by $\beta = 8\pi nk_B(T_p + T_e)/B_0^2 = v_{tp}^2(1 + T_e/T_p)/v_A^2$. In the limit of $k_z\rho_p \ll 1$ and $v_{ts}, v_A \ll c$, which we take here, the Alfven wave properties given in Figure 1 are independent of the exact values of $k_z\rho_p$ and v_{ts}, v_A , and depend only on λ_p , T_p/T_e , and β .

Figure 1a shows the parallel phase speed of the Alfven wave in units of the Alfven speed ($v_{\parallel}/v_A \equiv \text{Re}(\omega)/k_z v_A$). The two curves correspond to $T_p = T_e$ and $T_p = 10^3 T_e$. For $T_p \gtrsim 10T_e$, v_{\parallel} is nearly identical to the $T_p = 10^3 T_e$ result shown in the figure. In the MHD limit ($\lambda_p \ll 1$), we have $v_{\parallel} \simeq v_A$, the usual Alfven wave dispersion relation, while in the $\lambda_p \gg 1$ limit $v_{\parallel} \simeq v_A \sqrt{\lambda_p} \simeq v_A k_{\perp} \rho_p$, i.e., the wave frequency depends strongly on the perpendicular wave number. This result is well-known from analytic treatments in the $\beta \ll 1$ limit (e.g., Hasegawa & Chen 1976).

The remaining panels in Figure 1 specify properties of the dissipation of the Alfven wave by collisionless effects. Figure 1b gives the dissipation rate of the mode in units of the mode period, γT . Figure 1c gives the dimensionless proton, P_p , and electron, P_e , heating rates, using equation (7). For $T_p \gtrsim 10T_e$, the dimensionless proton heating rate, P_p , is nearly identical to the $T_p = 10^3 T_e$ result shown in the figure. We note that γT (Fig. 1b) can be derived from P_s (Fig. 1c) using $P_p + P_e = 2\gamma T$. Finally, in Figure 1d we explicitly show the relative heating of the protons and electrons, P_p/P_e .

As we are also interested in plasmas with $\beta \gtrsim 1$, Figure 2 shows γT (Fig. 2a) and P_p/P_e (Fig. 2b) for several β with $T_p/T_e = 100$. For $\beta \gtrsim 1$, the behavior of the dissipation of the Alfven wave with varying T_p/T_e is very similar to that shown in Figure 1 (the $\beta = 1$ case): γ is relatively independent of T_p/T_e for $\lambda_p \lesssim 1$, but the relative heating of protons and electrons (P_p/P_e) increases with increasing T_p/T_e , as in Figure 1d. This is explained analytically below.

For our purposes, the relevant parameter regime in wavevector space is $\lambda_p \lesssim 1$. This is because, as is discussed in more detail in §5.2, the turbulent dynamics of Alfven waves

due to GS is only valid in this limit. For $\lambda_p \lesssim 1$, the key qualitative results are that the Alfvén wave can be strongly damped by collisionless effects, particularly if $\beta \gtrsim 1$. Furthermore, provided that $T_p \gg T_e$, the dissipated wave energy primarily heats the protons. We note that, for $\lambda_p \gtrsim 1$, $v_{\parallel} \simeq v_A \sqrt{\lambda_p} \sim v_{tp} \sqrt{\lambda_p}$ and so with increasing λ_p there are progressively fewer protons available to resonate with the wave. This is why the proton contribution to the damping, P_p , falls off exponentially for $\lambda_p \gtrsim 1$ (Fig. 1c).

The dissipation properties of the Alfvén wave shown in Figures 1 and 2 can be understood in terms of primarily TTD by the protons; the generally smaller electron contribution is due to both LD and TTD for $\beta \simeq 1$, while TTD dominates for larger β .³ This can be seen by looking at the relative contributions of TTD and LD in the expression for the particle heating, P_s (eq. [7]). The TTD contribution is $\propto \chi_{yy}^a |E_y|^2$ (which is $\propto \chi_{yy}^a |B_z|^2$ by Faraday’s Law) while the LD term is $\propto \chi_{zz}^a |E_z|^2$. In Figure 3 we show, for $\lambda_p = 0.1$, the ratio of the TTD and LD contributions to the proton (Fig. 3a) and electron (Fig. 3b) heating rates as a function of T_p/T_e for several β ; below we discuss the physical/analytical origin of these results. We emphasize that, even for $\lambda_p \sim 1$, $\omega \ll \Omega_p$ (since $\omega \simeq k_z v_A$ and $k_z \rho_p \ll 1$ by assumption); the magnetic moment of a particle is thus an adiabatic invariant, as is required for TTD.

From equation (4), it is relatively straightforward to find the dependence of χ_s^a on β and T_p/T_e .⁴ For a fixed T_p/T_e , this yields $\chi_{yy}^a/\chi_{zz}^a \propto \beta$ for both protons and electrons, while for a fixed β we find that $\chi_{yy,e}^a/\chi_{zz,e}^a \propto (T_e/T_p)^2$ and that $\chi_{yy,p}^a/\chi_{zz,p}^a$ is independent of T_p/T_e . Physically, increasing β (at fixed T_p/T_e) increases the proton and electron thermal speeds with respect to the Alfvén speed, thus increasing the magnetic moment of the thermal particles. This makes TTD more important, which is why $\chi_{yy}^a/\chi_{zz}^a \propto \beta$. Increasing T_p/T_e (at fixed β), on the other hand, effectively decreases the magnetic moment of the electrons, which tends to decrease the contribution to electron heating from TTD. This is why $\chi_{yy,e}^a/\chi_{zz,e}^a \propto (T_e/T_p)^2$. The parallel electric field of the Alfvén wave, however, decreases with increasing T_p/T_e . In the $\beta \ll 1$ limit, $E_z \propto T_e/T_p$ (at fixed β), while E_y is independent of T_e/T_p (e.g., Hasegawa & Chen 1976; Melrose 1986, p. 178)⁵; we find that this is also roughly satisfied if $\beta \simeq 1$. These scalings imply that $\chi_{yy,p}^a |E_y|^2 / \chi_{zz,p}^a |E_z|^2 \sim \beta (T_p/T_e)^2$ and that $\chi_{yy,e}^a |E_y|^2 / \chi_{zz,e}^a |E_z|^2 \sim \beta$, which reproduce the numerical calculations reasonably well.

These considerations show that, to a good degree of accuracy, one can take $P_p/P_e \simeq \chi_{yy,p}^a/\chi_{yy,e}^a$, since both the electron and proton heating is primarily due to TTD. This greatly simplifies evaluating P_p/P_e analytically since a detailed expression for the electric

³For $\beta \simeq 1$ and $T_p \simeq T_e$ the Landau and transit time contributions are comparable for both electrons and protons.

⁴For the regime of interest here, one need only keep the $n = 0$ terms and the leading order λ_p terms.

⁵This is because E_y arises from keeping kinetic terms which are dropped in the MHD limit while E_z is due to both thermal and kinetic corrections.

field vector is not needed. Using equation (4) and the MHD Alfven wave dispersion relation, we find that

$$\frac{P_p}{P_e} \simeq \left(\frac{m_p T_p}{m_e T_e} \right)^{1/2} \exp \left[- \left(1 + \frac{T_e}{T_p} \right) \beta^{-1} \right], \quad (9)$$

where we have taken $v_{te} \gg v_A$ and $\lambda_p \lesssim 1$. The small deviations from equation (9) in Figures 1d and 2b (for $\lambda_p \lesssim 1$) are due to the contribution of LD to the electron heating. In the next section we give a more general version of equation (9) which is valid for any wave damped primarily by TTD.

Finally, we note that the results given here for the dissipation of nearly perpendicular Alfven waves in a $\beta \gtrsim 1$ plasma differ from those obtained by Stefant (1976), who found that the Alfven wave dissipation rate reaches a maximum ($\gamma T \simeq 0.1$) at some particular value of β and decreases for larger β . The reason for this discrepancy is straightforward. Stefant did not solve the full dispersion relation (eq. [3]), but instead used a simplified dispersion relation that neglected all contributions from the yy components of the susceptibility tensor.⁶ This amounts to considering only LD of the Alfven wave. This is valid only in the $\beta \ll 1$ limit when the magnetic compression of the Alfven wave, B_z , is negligible (e.g., Hasegawa & Chen 1976, who use a dispersion relation very similar to Stefant's, but explicitly state that it is only valid for small β). Our calculations agree with Stefant's in the $\beta \ll 1$ limit; including the yy susceptibilities, which are responsible for TTD, is, however, necessary in the $\beta \sim 1$ limit. In fact, in the limit of $\beta \gg 1$, Foote and Kulsrud (1979) have shown analytically, by expanding the susceptibility tensor to leading order in β^{-1} , that only TTD contributes to the dissipation of the Alfven wave.

3.4. General Relations for Particle Heating by TTD and LD

Equation (9) for the relative heating of protons and electrons in the dissipation of a nearly perpendicular Alfven wave is more general than it might appear. From the susceptibility tensor (eq. [4]) and the particle heating rate (eq. [7]) one can show that, for any wave which is damped solely by TTD, the relative heating of protons and electrons in the $\lambda_p \lesssim 1$ limit is

$$\left(\frac{P_p}{P_e} \right)_{TTD} \simeq \left(\frac{m_p T_p}{m_e T_e} \right)^{1/2} \exp \left[- \left(\frac{v_{\parallel}}{v_{tp}} \right)^2 + \left(\frac{v_{\parallel}}{v_{te}} \right)^2 \right] \simeq \left(\frac{m_p T_p}{m_e T_e} \right)^{1/2}. \quad (10)$$

⁶His dispersion relation also assumed $\lambda_p \ll 1$.

A similar analysis for LD shows that, for any wave which is damped solely by LD, the relative heating of protons and electrons in the $\lambda_p \lesssim 1$ limit is

$$\left(\frac{P_p}{P_e}\right)_{LD} \simeq \left(\frac{m_p T_e^3}{m_e T_p^3}\right)^{1/2} \exp \left[- \left(\frac{v_{\parallel}}{v_{tp}}\right)^2 + \left(\frac{v_{\parallel}}{v_{te}}\right)^2 \right] \simeq \left(\frac{m_p T_e^3}{m_e T_p^3}\right)^{1/2}. \quad (11)$$

The latter equalities in equations (10) and (11) correspond to either subthermal waves ($v_{\parallel} \lesssim v_{te}, v_{tp}$) or equal electron and proton thermal speeds ($v_{te} \simeq v_{tp}$). The physical origin of these relations is as follows. The LD expression can be written as $(P_p/P_e)_{LD} \simeq (m_e/m_p)(v_{te}^3/v_{tp}^3)$. The first term (m_e/m_p) is the relative acceleration of protons and electrons for a given force. The second term (v_{te}^3/v_{tp}^3) is the relative number of particles available to resonate with the wave (i.e., the relative slopes of the proton and electron distribution function at the wave’s phase speed, taking $v_{\parallel} \lesssim v_{te}, v_{tp}$). Equation (10) for TTD can be written as $(P_p/P_e)_{TTD} \simeq (m_e/m_p)(v_{te}^3/v_{tp}^3)(\mu_p^2/\mu_e^2)$, where $\mu_s \propto T_s$ is the magnetic moment of particles with the thermal velocity. The first two terms in the TTD expression are identical to the LD expression, and have the same physical interpretation. The last term reflects the fact that in TTD, the wave-particle interaction is a function of the particle’s magnetic moment. The larger μ_s , the stronger the coupling between the wave and the particle. In LD the corresponding term is $q_p^2/q_e^2 = 1$ since the wave-particle coupling is the electrostatic force.

Equation (10) shows that, quite generally, TTD is a natural mechanism for preferentially heating protons in plasmas with $T_p \gg T_e$. This is because in such plasmas the protons have the larger magnetic moment and so couple better to a wave’s magnetic field perturbation. LD, on the other hand, leads to preferential electron heating in plasmas with $T_p \gg T_e$.

3.5. Collisionless dissipation of the fast and slow MHD modes

A given excitation at the outer scale will, in general, contain both noncompressive (Alfvénic) and compressive (fast and slow mode) components. For completeness, we therefore briefly consider the collisionless dissipation of the fast and slow MHD modes in plasmas with $T_p \gg T_e$. Since these modes have either $B_z \neq 0$ or $E_z \neq 0$ in the MHD limit (and as there is no detailed theory of fast or slow mode turbulence to indicate if the cascade is parallel, perpendicular, or isotropic) we consider the dissipation only in the MHD limit. For $T_p \simeq T_e$ this problem has been considered in detail by Barnes (1966; 1967; 1968a; 1968b). Figure 4 shows the dimensionless dissipation rate, γT , of the fast (Fig. 4a) and slow (Fig. 4b) MHD modes as a function of θ , the angle between the wavevector and the background magnetic field, for several T_p/T_e for a $\beta = 1$ plasma (we take $k\rho_p \ll 1$ and $v_A, v_{ts} \ll c$). The $T_p = T_e$ results are identical to those of Barnes (1966).

It is well known that the fast mode is damped primarily by TTD (Barnes 1966; Miller 1991). This is because, in the MHD limit, the electric field vector of the wave is along $\hat{\mathbf{y}}$, which leads to a strong compressional magnetic field perturbation, B_z . The relative heating of the protons and electrons is thus given by equation (10) of the previous section:

$$\left(\frac{P_p}{P_e}\right)_{fast} \simeq \left(\frac{m_p T_p}{m_e T_e}\right)^{1/2} \exp \left[- \left(1 - \frac{T_p m_e}{T_e m_p}\right) \left(1 + \frac{T_e}{T_p}\right) \sec^2(\theta) \right], \quad (12)$$

where we have taken $\beta \simeq 1$ and have used the MHD dispersion relation $\omega \simeq kv_A$ (which is also reproduced by our kinetic theory calculations), so that $v_{\parallel} \simeq \sec(\theta)v_A$. In a one temperature plasma, electrons are preferentially heated by fast modes with $\theta \gtrsim 60^\circ$. This is because, at these angles, $v_{te} \gg v_{\parallel} \gg v_{tp}$ and there are no resonant protons, but plenty of resonant electrons (see also eq. [12]). In a $T_p \gg T_e$ plasma, however, there is no propagation angle for which the fast mode is strongly damped and the electrons are preferentially heated. This is because $v_{te} \sim v_{tp}$ and so the above condition on the wave's parallel phase speed cannot be obtained. This also follows directly from equation (12) by setting $v_{tp} \sim v_{te}$, in which case $(P_p/P_e)_{fast} \simeq (m_p T_p/m_e T_e)^{1/2}$, independent of θ . In the MHD regime, fast mode turbulence in plasmas with $T_p \gg T_e$ should therefore lead to primarily proton heating. There is, however, a large range of propagation angles ($\theta \gtrsim 60^\circ - 70^\circ$; see Fig. 4a) for which the fast mode is essentially undamped (as opposed to $\theta \gtrsim 88^\circ$ for a one temperature plasma). These modes would likely cascade out of the MHD regime.

The slow MHD mode is essentially a sound wave modified by the presence of a magnetic field; it thus has a large E_z and is Landau damped. For $T_p \gg T_e$ this leads to preferential electron heating (§3.4). As Figure 4b indicates, the slow mode is very strongly damped by collisionless effects; this is particularly true in a $T_p \gg T_e$ plasma, since there are more electrons available to resonate with the wave. This likely precludes slow mode turbulence cascading out of the MHD regime (again, particularly in a $T_p \gg T_e$ plasma).

4. The Effect of Wave Dissipation in a Perpendicular Alfvénic Cascade on the Electron and Proton Distribution Functions

Particle acceleration by Alfvénic turbulence has often been discussed utilizing either parallel or isotropic turbulent cascades (e.g., for solar flares, Miller & Roberts 1995; Smith & Miller 1995; for accreting black holes, Dermer et. al. 1996; Li et. al. 1996). In the case of an isotropic cascade, these works also neglect the $n = 0$ wave particle interactions (which are the focus of this paper). These assumptions allow a dramatic simplification of the quasi-linear diffusion equations, which describe the response of the

particles to the dissipated wave energy as a diffusion in velocity space (e.g., Melrose 1980). In contrast to the perpendicular cascade of Alfvén waves considered here, dissipation occurs when $\omega \sim \Omega_p$. This corresponds to the $|n| = 1$ resonance in the single particle resonance equation (eq. [1]). In this case the turbulent energy is transferred entirely to the protons, regardless of T_p/T_e (since the damping is due to a resonance between the proton’s cyclotron motion and the wave’s perpendicular electric field).

Dissipation of Alfvénic turbulence by the cyclotron resonance is attractive because it naturally leads to the formation of strong non-thermal features in the proton distribution function (and acceleration of protons to relativistic energies), which are suggested by observations of both solar flares and accreting black holes. To see qualitatively how this occurs, note that an isotropic Alfvénic cascade has waves with frequencies from the outer scale frequency up to $\sim \Omega_p$. From the resonance condition, equation (1), we see that waves with $\omega \simeq \Omega_p$ accelerate protons with $v_z \sim v_A \sim v_{tp}$ (we take $\beta \sim 1$ in this section). Waves with $\omega \lesssim \Omega_p$ can only accelerate particles if there is a big Doppler shift in the wave frequency, i.e., particles with $v_z \sim v_{tp}\Omega_p/\omega$. Thus a spectrum of waves with ω/Ω_p ranging from $\ll 1$ up to ~ 1 can naturally accelerate particles from thermal to relativistic energies. The above references contain detailed calculations of the evolution of the proton distribution function by this process.

The work of GS suggests, however, that isotropic Alfvénic cascades are unlikely to be obtained. As we noted in §3.1, when dissipation in a nearly perpendicular Alfvénic cascade occurs, $\omega \ll \Omega_p$. This implies that $n \neq 0$ resonances (such as the cyclotron resonance) are unimportant since there are a negligible number of particles with $v_z \sim v_{tp}\Omega_p/\omega \gg v_{tp}$. Since only $n = 0$ resonances are relevant, the resonance condition simplifies to $\omega = k_z v_z$ or, using the MHD Alfvén wave dispersion relation, $v_z = v_A \sim v_{tp}$. We emphasize that this condition, which is independent of the wave frequency and wavevector, holds for *all waves* in a nearly perpendicular Alfvénic cascade. The effect of the turbulent energy on the proton distribution function is thus qualitatively as follows. All waves dissipate their energy to particles with $v_z = v_A \sim v_{tp}$. This increases the *parallel* energy of particles near the peak of the thermal distribution, but no suprathermal feature is formed; there are simply no waves which can accelerate suprathermal particles. Only the parallel energy of the particles increases since, for $\omega \ll \Omega_p$, a particle’s magnetic moment is an adiabatic invariant; its perpendicular energy therefore cannot change. The resulting parallel proton distribution function is reasonably well approximated as thermal, since it is roughly monotonic and has a well-defined mean energy (e.g., Begelman & Chiueh 1988). For $T_p \gg T_e$, the same holds for electrons since then $v_{te} \sim v_A$ and the particles which are heated also reside near the peak of the thermal distribution function.⁷

⁷For $T_p \sim T_e$, the electron heating may lead to a “bump” in the distribution function at $v_z \simeq v_A \ll v_{te}$ (in addition to the thermal peak near v_{te}). The same holds for protons if β is much different from 1 since then $v_{tp} \neq v_A$. In these cases, there is still no acceleration of particles to relativistic energies,

The fact that TTD only heats the parallel energy of the particles is significant since the efficiency of TTD depends on the particle’s magnetic moment, which is proportional to the particle’s perpendicular energy. If E_{\perp} is unchanged as the plasma is heated the efficiency of TTD would be substantially reduced. In §6.4 we discuss mechanisms which lead to isotropy in the distribution function in hot accretion flows.

5. Applications to the GS cascade

5.1. Validity of the linear theory

For the linear analysis of the previous sections to be applicable, a particle’s motion must be reasonably well approximated by the guiding center approximation (free streaming along \mathbf{B}_0 and cyclotron motion perpendicular to \mathbf{B}_0). The presence of a turbulent cascade clearly perturbs a particle’s motion, so it is worthwhile examining to what extent the linear analysis is valid for the strong turbulent cascade of GS. First we consider motion parallel to the mean field, which is particularly important since we are interested in the $n = 0$ resonances, which occur between the free streaming motion of the particle and the wave’s parallel fields.

For large wavevectors the Alfvén wave has finite perturbed electric and magnetic fields along the mean magnetic field (E_z and B_z), which are responsible for the damping discussed in the previous sections. If the parallel fields are too large, however, they trap the particles in the potential fluctuations of the wave, violating the assumption of free streaming. This occurs within the cascade time provided that $\omega\tau_{e,b} \lesssim 1$, where $\tau_e \sim (m/qk_z E_z)^{1/2}$ is the characteristic oscillation period in the potential well of the parallel electric field (Stix 1992) and $\tau_b \sim (m/\mu k_z^2 B_z)^{1/2}$ is its analogue for the parallel magnetic field. From the numerical calculations, $E_z \sim E_x \lambda_p^{1/2} \eta T_e/T_p$ and $E_y \sim E_x \eta$ (taking $\beta \sim 1$; see also Melrose 1986, p. 178); using Faraday’s Law and the GS scaling for $B_y = B_k$ from §2,

$$E_z \sim B_{out} \frac{v_A}{c} \frac{T_e}{T_p} \lambda_p^{2/3} \left(\frac{\rho_p}{L} \right)^{2/3} \quad (13)$$

and

$$B_z \sim B_{out} \lambda_p^{1/3} \left(\frac{\rho_p}{L} \right)^{1/3}. \quad (14)$$

Even though the turbulent energy density (B_k^2) decreases with increasing \mathbf{k} (§2), the parallel field strengths increase with increasing \mathbf{k} because the kinetic theory corrections win out over the decreasing wave energy. Using equations (13) and (14), it is

although a potentially significant non-thermal (non-monotonic) feature in the distribution function may develop (depending on the ratio of the turbulent energy dissipated to the thermal energy near $v_z \simeq v_A$).

straightforward to check that, for the protons, $\omega\tau_e \sim \lambda_p^{-1/6}(L/\rho_p)^{1/6}(T_p/T_e)^{1/2} \gg 1$ and $\omega\tau_b \sim \lambda_p^{-1/6}(L/\rho_p)^{1/6} \gg 1$ (for the applications discussed in §6, $L/\rho_p \gtrsim 10^8$). Thus, the turbulence does not appear to contain sufficiently large parallel fields to trap the particles.

We now consider motion perpendicular to the mean field, which is more complicated. For a given wave in the GS cascade the $\mathbf{E} \times \mathbf{B}$ drift velocity is $\mathbf{v_E} = c\mathbf{E} \times \mathbf{B}/B^2 \sim v_A(k_\perp L)^{-1/3}(-\hat{\mathbf{y}} + E_y/E_x\hat{\mathbf{x}})$. Using the above scaling for E_y/E_x implies

$$\mathbf{v_E} \sim v_A \left[-\lambda_p^{-1/6} \left(\frac{\rho_p}{L} \right)^{1/3} \hat{\mathbf{y}} + \lambda_p^{1/6} \left(\frac{\rho_p}{L} \right)^{2/3} \hat{\mathbf{x}} \right]. \quad (15)$$

This leads to a Doppler shift in the frequency of the wave seen by the particle of $\mathbf{k} \cdot \mathbf{v_E} = k_\perp v_{E,x} \sim \omega \lambda_p^{1/3}(\rho_p/L)^{1/3}$, since \mathbf{k} is in the x-z plane (§3.2). In addition, the finite lifetime of Alfvén waves in the GS cascade introduces shifts in the mode frequency which are $\sim \omega$ (since the cascade time is of order the linear mode period).

Since the Doppler shifts due to the GS cascade are $\sim \omega \ll \Omega_p$, the magnetic moment of the particles is still an adiabatic invariant. This is important for the applicability of TTD. The single particle resonance condition (eq. [1]), however, shows up as a delta function in the linear theory. It therefore cannot be rigorously applicable to the GS cascade since there are additional frequency shifts $\sim \omega$ not accounted for in the linear theory. This leads to a broadening of the resonance condition, but only by a factor of order unity. The usual result of resonance broadening is that it makes more particles available to resonate with the wave. For our problem, this has little effect since in linear theory there are already a significant number of resonant particles – the parallel phase speed of the Alfvén wave is comparable to the proton and electron thermal velocities. Furthermore, while the turbulence is strong in a turbulence theory sense, the wave amplitudes are not so large as to broaden the resonance condition by many harmonics of the wave frequency, which would significantly reduce the particle heating (Begelman & Chiueh 1988).⁸ These considerations suggest to us that the linear analysis should be a good first approximation for the GS cascade. This does not, of course, preclude that other dissipation mechanisms neglected in the linear analysis could be important.

5.2. Resolution of the GS Cascade

In the GS cascade, the cascade time is of order the linear Alfvén period; conversion of a significant fraction of the turbulent energy to thermal energy via dissipation thus requires a damping time of order the mode period, i.e., $\gamma T \sim 1$. Furthermore, to

⁸This happens when the Doppler shifts are much greater than the mode frequency.

consistently apply the turbulent dynamics of GS, this dissipation must occur when the MHD approximations made in their calculation are reasonably applicable. This requires $k\rho_p \lesssim 1$ and $\omega \lesssim \Omega_p$, which simplify to $k_\perp\rho_p \lesssim 1$ for the nearly perpendicular cascade of GS.

From the calculations described in §3, it follows that (for $\beta \gtrsim 1$ plasmas) Alfvén waves with $k_\perp\rho_p \lesssim 1$ and $\omega \ll \Omega_p$ have damping rates satisfying $\gamma T \gtrsim 1$ only if $\beta \gtrsim 5$. In large β plasmas the proton magnetic moment couples better to the wave’s magnetic field perturbation, leading to more efficient TTD. If $\beta \gtrsim 5$, most ($\gtrsim 50\%$) of the turbulent energy in Alfvén waves is dissipated in the GS cascade. The relative heating of protons and electrons by this dissipated energy is given approximately by equation (9).

What happens, however, for the astrophysically important case of a $\beta \sim 1$ plasma? Proton heating still dominates over electron heating, but Figure 1 gives $\gamma T \simeq 0.1$ for an Alfvén wave with $k_\perp\rho_p \sim 1$, so that only a small fraction of the Alfvénic energy is dissipated. TTD is not strong enough to damp the waves before they cascade out of the MHD regime. We can envision three possibilities for how the energy is ultimately dissipated, but are unable to ascertain which is realized.

1. Other dissipation mechanisms are important and dissipate the wave energy before $k_\perp \gtrsim \rho_p^{-1}$.

2. The cascade continues past $k_\perp \sim \rho_p^{-1}$, but how it does so (i.e., along what track in \mathbf{k} space) and in which modes the energy resides is unknown; how the turbulent energy is ultimately dissipated is thus unknown. The structure of the GS cascade is crucially dependent on the polarization (i.e., the velocity eigenfunction) and dispersion relation of the Alfvén wave. Both of these properties are entirely different for $k_\perp\rho_p \gtrsim 1$ (the kinetic limit) than they are in the GS regime ($k_\perp\rho_p \lesssim 1$). For example, the wave frequency is $\propto k_\perp$ in the kinetic limit while it is independent of k_\perp in the GS regime. In the GS regime, the electrons and protons have the same velocity structure (the $\mathbf{E} \times \mathbf{B}$ drift is independent of q and m), while in the kinetic limit, the electrons move substantially faster than the protons;⁹ a single fluid analysis is thus no longer applicable. It is therefore unclear whether the turbulent energy will stay in the “Alfvén” wave in the kinetic limit. Furthermore, GS argue that, in the $k_\perp\rho_p \lesssim 1$ limit, the Alfvén wave in the nearly perpendicular cascade is poorly coupled to the slow and fast modes; this conclusion also rests on the polarization and dispersion relations of the modes and is thus inapplicable in the kinetic limit. For $k_\perp\rho_p \gtrsim 1$, the Alfvén wave therefore may (or may not) efficiently couple to fast or slow (or other) modes. Understanding this possibility is clearly a rather complicated problem in plasma turbulence.

3. As a result of poor coupling to other waves, the energy is dissipated at $k_\perp\rho_p \sim 1$. As

⁹This is because the wave’s electric field averages out over the proton Larmor orbit, but not the electron’s; the protons thus see an effectively smaller electric field and move more slowly as a result.

in the previous possibility, the energy “wants” to cascade to higher k_{\perp} , but being poorly coupled to any other modes, it can’t; the cascade time and mode energy thus increase (maintaining a constant energy flux) until the waves are dissipated.

6. Applications to Hot, Two Temperature, Accretion Flows

6.1. General Considerations

Angular momentum transport in thin accretion disks is now believed to arise from a magneto-rotational MHD instability discussed extensively by Balbus and Hawley (1991, hereafter BH; for a recent review, see Balbus & Hawley 1997). This instability has been considered primarily in the MHD limit, and its applicability to nearly collisionless systems is perhaps unclear. In Appendix B we argue (but do not prove) that the instability should proceed in collisionless systems, provided that the particle distribution functions are close to thermal. The BH instability, when it reaches nonlinear amplitudes, is large scale ($k \sim H^{-1} \sim R^{-1}$, where k is the wavevector of the instability, H is the disk scale height, R is the local radius in the accretion flow, and the latter equality is for quasi-spherical accretion flows) and so naturally couples to long wavelength waves, generating MHD turbulence; this is seen in numerous numerical simulations (e.g., Stone et al. 1996). This is the basic reason for supposing that most of the gravitational potential energy released by viscosity resides in MHD turbulence.¹⁰ In this paper we have made the further simplification of focusing primarily on Alfvénic turbulence; this is both because Alfvénic turbulence is (comparably) well understood and because some such restriction is a necessary first step in attempting to understand the problems considered in this paper. MHD turbulence generated by the BH instability may in fact be predominantly Alfvénic since the instability is, to linear order, noncompressive. This suggests that the excitation of compressive MHD turbulence (such as fast and slow modes) may be less important since it is a higher order nonlinear effect than the excitation of noncompressive (Alfvén) modes.¹¹

Advection dominated accretion flows (ADAFs) are the only known thermally and viscously stable, dynamically consistent models of hot, two temperature, accretion flows (Kato et al. 1996). For this reason, we frame the discussion in this section in terms of ADAFs. Much of what we say, however, will apply to any hot, two temperature,

¹⁰More generally, any large scale instability in the plasma will generate significant levels of MHD turbulence.

¹¹The Alfvén wave in a compressible medium is incompressive to linear order, but compressive when nonlinear effects are included (Holwegg 1971).

accretion flow. For the present purposes, the relevant properties of ADAFs are well described by the self similar solution of Narayan & Yi (1994; 1995b), which yields

$$\begin{aligned}
 \frac{v}{c} &\simeq 0.37 \alpha r^{-1/2}, \\
 T_p &\simeq 2 \times 10^{12} \frac{\beta}{\beta + 1} r^{-1} \text{ K}, \\
 n &\simeq 6.3 \times 10^{19} \alpha^{-1} m^{-1} \dot{m} r^{-3/2} \text{ cm}^{-3}, \\
 B &\simeq 10^9 \alpha^{-1/2} (\beta + 1)^{-1/2} m^{-1/2} \dot{m}^{1/2} r^{-5/4} \text{ Gauss}, \\
 \Omega_p &\simeq 10^{13} \alpha^{-1/2} (\beta + 1)^{-1/2} m^{-1/2} \dot{m}^{1/2} r^{-5/4} \text{ rad s}^{-1}, \\
 \frac{\rho_p}{R} &\simeq 6 \times 10^{-9} \beta^{1/2} \alpha^{1/2} m^{-1/2} \dot{m}^{-1/2} r^{-1/4}, \tag{16}
 \end{aligned}$$

where v/c is the radial velocity in units of the velocity of light, n is the number density of electrons/protons, B is the magnetic field strength, determined by assuming a constant β in the accretion flow¹², Ω_p is the proton cyclotron frequency, ρ_p is the Larmor radius of thermal protons, m is the mass of the central object in solar mass units, $M = m M_\odot$, \dot{m} is the accretion rate in Eddington units, $\dot{M} = \dot{m} \dot{M}_{\text{Edd}}$ ($\dot{M}_{\text{Edd}} = 1.39 \times 10^{18} m \text{ g s}^{-1}$), and $r = R/R_S$ is the radius in Schwarzschild units ($R_S = 2.95 \times 10^5 m \text{ cm}$). In equation (16), the fraction of the viscously dissipated energy that is carried inward by the accreting gas is taken to be ~ 1 .

The electron temperature in ADAF models typically saturates at $T_e \sim 10^9 - 10^{10} \text{ K}$ in the inner $10^2 - 10^3$ Schwarzschild radii since the efficient cooling of relativistic electrons prevents higher temperatures. Thus the electrons and protons are both marginally relativistic and the non-relativistic analysis employed in this paper is a reasonable first approximation. Using equation (16) we can compute the characteristic frequency (ν) and (field-free) mean free path (ℓ) for proton-proton (pp) and electron-electron (ee) Coulomb collisions (Mahadevan & Quataert 1997):

$$\nu_{pp} \simeq 30 \alpha^{-1} \left(\frac{\beta + 1}{\beta} \right)^{3/2} m^{-1} \dot{m} \text{ Hz}, \tag{17}$$

$$\nu_{ee} \simeq 10^8 \alpha^{-1} m^{-1} \dot{m} r^{-3/2} T_9^{-3/2} \text{ Hz}, \tag{18}$$

$$\frac{\ell_{pp}}{R} \simeq 10^3 \alpha \left(\frac{\beta}{\beta + 1} \right)^2 \dot{m}^{-1} r^{-3/2}, \tag{19}$$

$$\frac{\ell_{ee}}{R} \simeq 10^{-3} \alpha \dot{m}^{-1} r^{1/2} T_9^2, \tag{20}$$

where T_9 is the electron temperature in units of 10^9 K . ν_{ee} (ν_{pp}) is the rate at which the energy and direction of an electron (proton) changes appreciably through Coulomb

¹²The “ β ” used in papers on ADAF models, β_{adv} , is taken to be the ratio of the gas pressure to the total pressure and is thus related to the plasma physics β used in this paper by $\beta_{\text{adv}} = \beta/(\beta + 1)$.

collisions with the same particle species. Electron-proton collisions change the proton energy at a rate $\simeq \nu_{ee}m_e/m_p$. ℓ is the mean free path in the absence of a magnetic field. Since ℓ is often $\gtrsim R$, this is indicative of the collisionless nature of the plasma in ADAFs.

The characteristic frequency at the outer scale of the accretion flow, which is roughly the frequency at which waves (both Alfven and slow and fast MHD modes) will be excited, is $\omega_{out} \simeq v_A R^{-1} \simeq 6 \times 10^4 (\beta + 1)^{-1/2} m^{-1} r^{-3/2}$ rad s $^{-1}$. Comparing this with ν_{pp} , we see that the protons are effectively collisionless for all perturbations of interest (those with frequencies $\gtrsim \omega_{out}$). By contrast, however,

$$\nu_{ee}/\omega_{out} \sim 10^3 \alpha^{-1} (\beta + 1)^{1/2} \dot{m} T_9^{-3/2}. \quad (21)$$

Provided that

$$\dot{m} \gtrsim 10^{-3} \alpha (\beta + 1)^{-1/2} T_9^{3/2}, \quad (22)$$

$\nu_{ee} \gtrsim \omega_{out}$ and the electrons must be treated as *collisional* on the outer scale. This is probably not a significant complication for Alfven waves, since collisionless effects are unimportant at the outer scale. It must, however, be taken into account in treatments of the fast and slow modes, which undergo collisionless dissipation even in the MHD limit (§3.5). The fast mode is damped primarily by the protons. The net damping of the wave will therefore not be significantly modified by the collisionality of the electrons. The slow mode, on the other hand, is a modified sound wave which is strongly Landau damped by electrons in a collisionless plasma with $T_p \gg T_e$. The collisionality of the electrons for perturbations with $\omega \sim \omega_{out}$ will suppress the electron contribution to the damping and thus remove this source of electron heating (for accretion rates satisfying eq. [22]). The slow mode will, however, still be strongly damped by other mechanisms at the outer scale, e.g., Landau damping by the protons and wave steepening leading to (collisionless) shocks.

For the GS cascade, strong damping of the Alfven wave occurs when $k_\perp \simeq \rho_p^{-1}$, at which point $k_z \simeq \rho_p^{-2/3} R^{-1/3}$ and $\omega \equiv \omega_{in} \simeq \Omega_p \beta^{-1/2} (\rho_p/R)^{1/3}$. Using equation (16) and comparing ω_{in} with ν_{ee} , we find that

$$\omega_{in}/\nu_{ee} \simeq 10^2 \alpha^{2/3} \beta^{-1/2} (\beta + 1)^{-1/2} m^{1/3} \dot{m}^{-2/3} r^{1/6} T_9^{3/2}. \quad (23)$$

Thus, in the dissipation range, collisionless theory can be consistently applied for both electrons and protons. This conclusion is strengthened by noting that $m \gtrsim 1$ and $\dot{m} \lesssim 1$.

6.2. Collisional Dissipation of Alfven Waves

The (field free) mean free path for Coulomb collisions in a hot accretion flow is a significant fraction of the local radius. This might suggest that dissipation of waves by

microscopic viscosity could be important. The velocity fluctuation of an Alfvén wave is, however, perpendicular to the local magnetic field and so only cross field components of the viscosity tensor are important (GS). These are smaller than the field free component by a factor of $\sim (\nu_{pp}/\Omega_p)^2 \ll 1$, making microscopic viscous dissipation unimportant. More concretely, the dissipation range of a turbulent cascade set by microscopic viscous effects occurs at a scale $r_v \sim R/Re^{3/4}$, where the Reynold's number is $Re \sim v_A R/\mu_\perp$ and $\mu_\perp \simeq 0.1n/(T_p^{1/2}B^2)$ cm⁻² s⁻¹ is the cross field kinematic viscosity coefficient (e.g., Spitzer 1961). Using equation (16) we find that $Re \sim 10^{21}\beta^{1/2}(\beta+1)^{-2}mr^{-5/4}$ and so

$$r_v/\rho_p \sim 10^{-7}\alpha^{-1/2}\beta^{-7/8}(\beta+1)^{3/2}m^{-1/4}\dot{m}^{1/2}r. \quad (24)$$

Microscopic viscous effects thus become important only on negligibly small scales. An analogous result holds for thermal conductivity.

Finite electrical resistivity also leads to wave damping, which becomes important at a scale $r_{\eta_e} \sim R/(Re_m)^{3/4}$, where $Re_m \sim v_A R/\eta_e$ is the Magnetic Reynold's number at the outer scale and $\eta_e \simeq 10^{13}/T_e^{3/2} \sim T_9^{-3/2}$ cm² s⁻¹ is the electrical resistivity (Spitzer 1961). Using equation (16) yields $Re_m \sim 10^{16}\beta^{-1/2}mr^{-1/2}T_9^{3/2}$ and so resistive damping occurs when

$$r_{\eta_e}/\rho_p \sim 10^{-4}\beta^{-1/8}m^{-1/4}\dot{m}^{1/2}r^{1/8}T_9^{-9/8}. \quad (25)$$

Collisionless dissipation sets in on scales $\sim \rho_p$, which is \gg than the collisional dissipation scales considered in this section; as the energy cascades to small wavelengths, it will therefore first encounter collisionless dissipation processes.

6.3. Implications of Particle Heating for ADAFs

It is usual in ADAF models to specify the relative heating of protons and electrons by a parameter δ , the fraction of the viscous energy which heats the electrons. In order for the optically thin ADAF formalism to be relevant to an accretion flow, one of two physical situations must occur:

1. $\delta \lesssim 0.5$ and $\dot{m} \lesssim \alpha^2$. In this case a significant fraction of the viscous energy is transferred to the protons; by virtue of the low accretion rate (which implies low densities; see eq. [16]), Coulomb collisions are too inefficient to transfer this energy to the electrons in the inflow time of the gas (Rees et al. 1982). Consequently, a fraction $\sim 1 - \delta$ of the viscous energy is advected, by the protons, onto the central object.
2. $\delta \sim 1$ and $\dot{m} \lesssim 10^{-4}\alpha^2$. In this case most of the viscous energy heats the electrons; the flow can be advection dominated only if the electron cooling time (the time for the electrons to radiate their thermal energy) is longer than the inflow time of the gas. For synchrotron cooling, the most efficient cooling mechanism in ADAFs at low accretion rates (Narayan & Yi 1995b), this occurs for $\dot{m} \lesssim 10^{-4}\alpha^2$ (Mahadevan & Quataert 1997).

The contribution to δ from the collisionless dissipation of Alfvén waves considered in §3 is (eq. [9])

$$\delta \equiv \frac{P_e}{P_p + P_e} \sim \left(\frac{m_e T_e}{m_p T_p} \right)^{1/2} \exp[(1 + T_e/T_p)\beta^{-1}]. \quad (26)$$

For accretion flows with $T_p \gg T_e$, this contribution is $\ll 1$. For most ($\gtrsim 50\%$) of the energy in Alfvénic turbulence to be dissipated by the mechanisms considered in this paper, however, the accretion flow must have $\beta \gtrsim 5$ (§5.2), which is different from the value of $\beta \sim 1$ usually used in ADAF models. Furthermore, numerical simulations of MHD turbulence in thin disks suggest that α and β are coupled, with $\alpha \simeq 0.5/(\beta + 1)$ (Hawley, Gammie, & Balbus 1996; Table 4). In this case $\beta \gtrsim 5$ would correspond to $\alpha \lesssim 0.1$, which is smaller than the value of $\alpha \simeq 0.25$ usually used in ADAF models. This $\alpha - \beta$ relationship may not, however, be applicable to ADAFs, which are *radially* convective and thus have a purely hydrodynamic source of angular momentum transport.

ADAF models typically take $\delta \sim m_e/m_p \sim 10^{-3}$ (e.g., Narayan et al. 1997), but they are relatively insensitive to δ so long as $\delta \ll 1$. The reason is that, in this limit, viscous heating of electrons is not their dominant heating mechanism. This can be seen by considering the energy equation for electrons in a hot accretion flow,

$$\rho T v \frac{ds}{dR} = \rho v \frac{d\epsilon}{dR} - q^c = q^{e+} - q^-, \quad (27)$$

$$q^c \equiv k T v \frac{dn}{dR},$$

$$q^{e+} = q^{ie} + q^v,$$

where s is the entropy of the electrons per unit mass of the gas, ϵ is the internal energy of the electrons per unit mass, q^c is the compressive heating (or cooling) rate per unit volume, and q^- is the energy loss due to radiative cooling. The total external heating of the electrons, q^{e+} , is a sum of the heating via Coulomb collisions with the hotter protons, q^{ie} , and direct viscous heating, q^v .

For accretion rates such that

$$\dot{m} \gtrsim 10^{-4} \alpha^2 \left(\frac{\delta}{10^{-3}} \right) T_9^{3/2}, \quad (28)$$

Coulomb heating of the electrons dominates over viscous heating (Mahadevan 1997), and so the precise value of δ is unimportant. The ratio of compressive to viscous heating is given roughly by (Mahadevan & Quataert 1997)

$$q^c/q^v \sim 10^{-3} \delta^{-1} T_9 r. \quad (29)$$

For small δ , compressive heating of the electrons is more important than viscous heating, again making the precise value of δ unimportant. For most of the systems to which

ADAF models have been applied, \dot{m} is sufficiently high and compressive heating is sufficiently important that only if δ were $\gtrsim 0.03$ in the inner regions of the accretion flow ($r \sim 3 - 100$, where the observed radiation originates) would the models be significantly modified.¹³

6.4. Isotropy?

The mechanisms for dissipating Alfvénic turbulence considered in this paper heat only the component of a particle’s energy which is parallel to the local magnetic field (§4). In our calculations, however, we have assumed an isotropic distribution function, which must be justified. A standard isotropization mechanism for both electrons and protons is pitch angle scattering. This can occur much more efficiently than particle heating if the parallel phase speed of the waves responsible for the scattering is much less than the particle’s speed (Melrose 1980); in this case isotropy could be maintained without significant heating. While this is a plausible mechanism for maintaining isotropy in hot accretion flows, we have not investigated this mechanism in detail.

Rather, we wish to point out a mechanism, unique to ADAFs, which maintains rough isotropy in the proton distribution function provided that the turbulent heating mechanism affects only the parallel component of the proton’s energy. In addition to heating by the viscously generated energy, particles are heated by compression as they accrete inwards. Since the particles are tied to the field lines, which are being compressed, adiabatic invariance of the particle’s magnetic moment requires $E_{\perp} \propto B$, where E_{\perp} is the particle’s perpendicular energy. For protons in an ADAF, the viscous heating rate is comparable to the compressional heating rate; this is because most of the viscous energy is stored as thermal energy of the protons and so all terms in the proton entropy equation are of the same order. If the viscous heating mechanism heats primarily the parallel energy of the protons, as the wave-particle interactions considered in this paper do, then the rough equality of viscous and compressional heating rates implies a rough equality of parallel and perpendicular heating, so that isotropy is maintained. Furthermore, Mahadevan & Quataert (1997) have shown that adiabatic compression maintains a thermal distribution of particles even in a collisionless gas (provided the particles are not compressed to relativistic energies). Therefore, given a thermal distribution of protons at large radii in an accretion flow, the perpendicular component of the distribution function remains thermal as they accrete onto the central object.

¹³An important exception to this is the Narayan et al. (1997) model of Sagittarius A*. For this system, the data is so good and the estimated accretion rate is so low ($\dot{m} \sim 10^{-4}$) that increasing δ to $\gtrsim 10^{-2}$ is problematic (see Fig. 4 of their paper).

This mechanism will not work for electrons since there is no necessary relationship between the compressive and viscous heating rates (the electrons are not, in general, advection dominated). For accretion rates satisfying equation (22), however, Coulomb collisions can maintain isotropy in the electron distribution function. This is because the characteristic timescale on which Alfvénic turbulence modifies the proton and electron distribution functions is $\sim \omega_{out}^{-1}$.¹⁴ Since the electrons (but not the protons) are effectively collisional on this timescale, isotropy is maintained.

6.5. Comments on related work

In a recent paper, Bisnovatyi-Kogan & Lovelace (1997; hereafter BL) suggest a mechanism which they feel leads to preferential electron heating in hot accretion flows. It is worthwhile examining this in some detail.

Throughout this paper we have argued that, on a microscopic level, particle heating occurs when the MHD turbulence generated at large scales in the accretion flow cascades to small scales and is dissipated by collisionless effects. BL seem to disregard this possibility, instead claiming that because the dissipation scale set by the hydrodynamic or magnetic Reynold’s number is extremely small, plasma instabilities set in which dissipate the turbulent energy. Collisionless effects, however, become important on length scales well above those set by the hydrodynamic or magnetic Reynold’s number (§6.2), obviating the need for an appeal to plasma instabilities.

BL argue that, on a microscopic level, particle heating in ADAFs is due to electric fields parallel to the local magnetic field accelerating particles to runaway velocities. By virtue of their smaller mass, electrons are more efficiently accelerated. In MHD, however, the electric field is given by $\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c + \eta_e \nabla \times \mathbf{B}/4\pi c$. In a highly conducting plasma, the magnitude of a typical electric field is $\sim vB/c$. Furthermore, the electric field is primarily perpendicular to the local magnetic field and thus unimportant for accelerating particles. In MHD parallel electric fields arise only from finite resistivity corrections and are $\sim vB/cRe_m$; since $Re_m \gg 1$ they are far too small to significantly accelerate particles in hot accretion flows.

Following Bisnovatyi-Kogan & Ruzmaikin (1976), however, BL argue that, because the flow is turbulent, one should use a “turbulent resistivity” (of order the Shakura-Sunyaev turbulent viscosity) instead of the usual microscopic resistivity; in this case, they argue, $E_{\parallel} \sim vB/c$ and parallel electric fields can significantly accelerate particles.

¹⁴This can be estimated from the quasi-linear diffusion equations, using the results of §5.1 (i.e., it is only the parallel fields, which arise from kinetic corrections to GS’s MHD cascade, which lead to diffusion of the particles in velocity space).

Our concern with this analysis is that, while the notion of a turbulent transport coefficient may be useful in describing the large scale properties of the flow (the global transport of angular momentum, for example), it should not be used on a microscopic level. When considering microscopic processes such as particle acceleration, the global transport processes in the flow are unimportant.

BL’s electron heating mechanism amounts to the claim that, on a microscopic level, the local electric field is *significantly aligned* with the local magnetic field. We see no reason, however, why the MHD results described above (the local electric field *perpendicular* to the local magnetic field) should be inapplicable to hot accretion flows.

7. Summary and Discussion

Hot accretion flows, such as advection dominated accretion flows (ADAFs), are effectively collisionless for all but the largest scale, lowest frequency, motions of the plasma (§6.1). Collective plasma effects are thus likely to be important in these systems. This is particularly true for particle heating since microscopic viscosity, thermal conductivity, and electrical resistivity are important only at extremely small scales. Particle heating influences both the global structure of the flow (e.g., by determining if the protons or electrons are heated) as well as the observed radiation (e.g., by determining the particle distribution functions).

Particle heating is a particularly important issue for ADAF models. All ADAF models which have been applied to observed systems assume that the viscously generated energy primarily heats the protons. This enables the accretion flow to be advection dominated so long as the timescale for electrons and protons to exchange energy by Coulomb collisions is longer than the inflow time of the gas. If viscosity *only* heats the electrons, an (optically thin) accretion flow can be advection dominated only when the electron cooling time is longer than the inflow time, which occurs at such low accretion rates that ADAF models would probably be less relevant for observed systems. Recent work by Bisnovatyi-Kogan & Lovelace (1997) suggests that all of the viscous energy in hot accretion flows does heat the electrons. In §6.5 we have argued against this conclusion.

In this paper we have focused on one aspect of the particle heating problem, namely particle heating by the linear collisionless dissipation of MHD, in particular Alfvén waves. We find it likely that the viscous energy in hot accretion flows resides primarily in MHD turbulence, making this mechanism of particular importance. An investigation similar to ours, but with somewhat different conclusions, was carried out independently by Gruzinov (1997).

Long wavelength Alfvén waves excited on the outer scale in hot accretion flows are undamped by linear collisionless effects. The nature of Alfvénic turbulence is thus crucial for assessing Alfvén wave damping and particle heating. Recent work on Alfvénic turbulence by Goldreich and Sridhar (1995; GS) suggests that the turbulent energy cascades almost entirely perpendicular to the local magnetic field (i.e., in the inertial range, $k_{\perp} \gg k_z$). As a result, the cyclotron resonance, which is usually thought to be significant in dissipating Alfvénic turbulence, is unimportant (since the wave frequencies are always much less than the proton cyclotron frequency).

We have shown, using the full kinetic theory dispersion relation for linear perturbations to a plasma, that Alfvén waves with frequencies much less than the proton cyclotron frequency, but with perpendicular wavelengths of order the Larmor radius of thermal protons, are damped by transit time damping (TTD) in plasmas appropriate to hot accretion flows ($T_p \gtrsim T_e$ and β , the ratio of the gas pressure to the magnetic pressure, $\gtrsim 1$). TTD is due to particles being accelerated by gradients in a wave’s longitudinal magnetic field perturbation, and is the magnetic analogue of Landau damping. For $T_p \gg T_e$, TTD quite generally leads to most of the dissipated wave energy heating the protons, since they have the larger magnetic moment and so couple better to the wave’s magnetic field perturbation (§3.3 and §3.4). For example, if $T_p \simeq 100T_e$, as is the case in the interior of ADAF models, the electrons receive only $\sim 2\%$ of the Alfvén wave energy as it is damped in a $\beta \sim 1$ plasma.

The dissipation rate of the Alfvén wave increases with increasing β since the proton’s magnetic moment is effectively larger and the wave-particle coupling is stronger (Figure 2; §3.3). For $\beta \gtrsim 5$, the dissipation of Alfvén waves by TTD is sufficiently strong to convert most ($\gtrsim 50\%$) of the Alfvénic energy to thermal energy before the MHD approximations utilized in the GS analysis cease to be valid. In the GS cascade, however, the energy travels so quickly through the inertial range that, for $\beta \sim 1$, TTD is not strong enough to dissipate the turbulent energy. In this case, the GS analysis of the turbulent dynamics breaks down before most of the turbulent energy is dissipated, and it is unclear to us how the cascade proceeds; how the Alfvénic energy is ultimately dissipated in a $\beta \sim 1$ plasma therefore remains unresolved by our work. In §5.2 we have enumerated (qualitatively) what we take to be the plausible possibilities.

So long as a significant fraction ($\gtrsim 50\%$) of the viscous energy heats the protons, the ADAF formalism is capable of describing the structure of an accretion flow (§6.3). This work suggests that, for $\beta \gtrsim 5$, Alfvénic turbulence in hot, two temperature, accretion flows preferentially heats the protons. Since we expect a significant fraction of the viscously generated energy to reside in Alfvénic turbulence, this alone can plausibly lead to greater than $\sim 50\%$ of the total viscously generated energy heating the protons.

Wave-particle interactions are often suggested as a mechanism for forming strong nonthermal features in the electron and proton distribution functions. In a perpendicular Alfvénic cascade of the kind proposed by GS, however, the wave damping is always due

to particles with velocities equal to the Alfvén speed (§4). In linear theory, it is thus impossible to accelerate particles to supra-Alfvénic velocities. For $\beta \sim 1$ and $T_p \gg T_e$, the Alfvén speed is near the thermal peak of the proton and electron distribution functions, so that the dissipated turbulent energy does not significantly modify the distribution functions from a Maxwellian. Perpendicular Alfvénic turbulence is therefore *not* a plausible mechanism for producing power law features in the proton and electron distribution functions or for accelerating particles to relativistic energies. This may exclude Alfvénic turbulence as a viable mechanism for particle acceleration in solar flares and accretion disk corona.

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A. Definitions of Some Oft-used Quantities

Quantity	Definition	Meaning
ω		mode frequency
T	$2\pi/\text{Re}(\omega)$	mode period
γ	$\text{Im}(\omega)$	mode damping rate
\mathbf{k}		mode wavevector
Ω_p	$qB/m_p c$	proton cyclotron frequency
v_{ts}	$\sqrt{2k_B T_s/m_s}$	proton (s = p) and electron (s = e) thermal speeds
ρ_p	v_{tp}/Ω_p	Larmor radius of thermal protons
λ_p	$0.5k_\perp^2 \rho_p^2$	dimensionless perpendicular wavevector
η	$k_z \rho_p$	dimensionless parallel wavevector
α		Shakura-Sunyaev viscosity parameter
β	$8\pi n k_B (T_p + T_e)/B^2$	ratio of gas to magnetic pressure
P_s		proton (s = p) and electron (s = e) heating rates
δ		fraction of viscous energy heating electrons

B. The Balbus-Hawley instability in a collisionless gas

There are three points which suggest to us that the MHD instability of Balbus & Hawley (BH) should apply to collisionless systems. While suggestive, these do not, of course, constitute a proof. The primary caveat to these comments is that, if the particle distribution functions are highly nonthermal, all bets are off.

1. Perhaps the primary concern in passing from the collisional to the collisionless version of an instability is that collisionless dissipation mechanisms may inhibit the instability. To linear order the axisymmetric version of the BH instability is, however, non-compressive and Alfvénic in character. As discussed in §3.1, in the MHD limit Alfvén waves are undamped by linear collisionless effects, which suggests that the instability should not be inhibited.

2. The collisionless limit entails the infinite conductivity limit used in ideal MHD; finite resistivity effects are particularly unimportant in a collisionless plasma.

3. The Keplerian rotation frequency, which is the characteristic growth rate of the instability, is $\Omega_o \simeq 7 \times 10^4 m^{-1} r^{-3/2}$ rad s⁻¹. The smallest characteristic frequency in a plasma is typically the proton cyclotron frequency, which is given in equation (16). For $\beta \ll 10^{16}$, $\Omega_o \ll \Omega_p$ and so the particles are tied to the field lines, which is a requirement for the instability to function. In the limit of $\Omega_o \gtrsim \Omega_p$ it is unlikely that the instability will persist. This corresponds, however, to exceedingly small magnetic field strengths ($\beta \gtrsim 10^{16}$).

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Figure Captions.

Figure 1: Properties of the Alfven wave as a function of $\lambda_p = 0.5k_\perp^2\rho_p^2$ for $\omega \ll \Omega_p$; various proton to electron temperature ratios, T_p/T_e , are considered for a plasma with equal gas and magnetic pressure ($\beta = 1$). (a) The parallel phase speed in units of the Alfven speed. For $T_p \gtrsim 10T_e$, v_\parallel is nearly the same as for $T_p = 10^3T_e$. (b) The dissipation per mode period, γT . (c) The dimensionless proton (solid line) and electron (dotted line) heating rates (P_s). T_p/T_e is shown along side each curve. For $T_p \gtrsim 10T_e$, the proton heating rate, P_p , is nearly identical to the $T_p = 10^3T_e$ case. (d) The relative proton and electron heating rates, P_p/P_e .

Figure 2: Properties of the Alfven wave as a function of $\lambda_p = 0.5k_\perp^2\rho_p^2$ for $\omega \ll \Omega_p$; various ratios of gas pressure to magnetic pressure (β) are considered for a $T_p = 100T_e$ plasma. (a) The dissipation per mode period, γT . (b) The relative proton and electron heating rates, P_p/P_e .

Figure 3: The relative contribution of transit time damping ($\chi_{yy}^a|E_y|^2$) and Landau damping ($\chi_{zz}^a|E_z|^2$) to the (a) proton and (b) electron heating rates as a function of the proton to electron temperature ratio (T_p/T_e); several β are considered, taking $\lambda_p = 0.1$.

Figure 4: The fractional dissipation per mode period, γT , in the MHD limit for (a) the fast mode and (b) the slow mode as a function of the angle between the wavevector and the background magnetic field (θ); various T_p/T_e are considered for a $\beta = 1$ plasma. For the slow mode, the curves are nearly vertical displacements of each other, so fewer T_p/T_e are shown.

